Bayesian grey-box identification of convection effects in heat transfer dynamics

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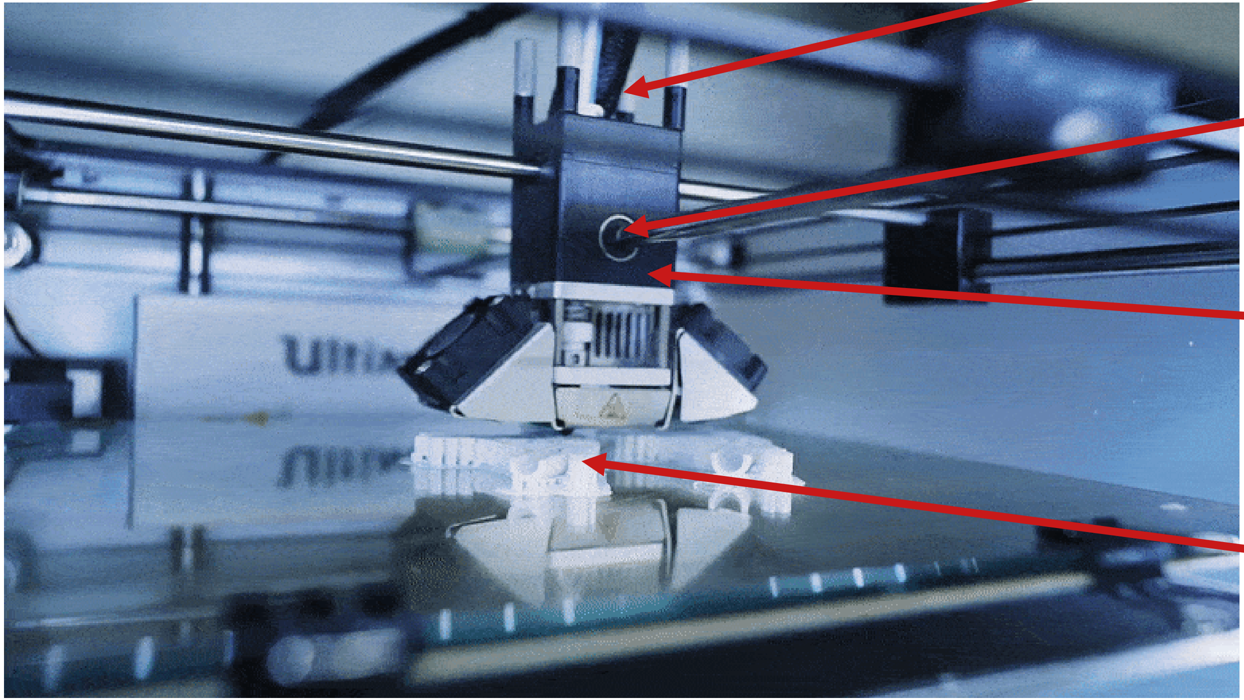
High-precision motion control



Molten material passes through

Actuator rods heat up

Metal deforms under heat

Deformed rods cause positioning errors

Image: 3Distrib https://www.3distrib.fr/en/kiosk-3d-printer/

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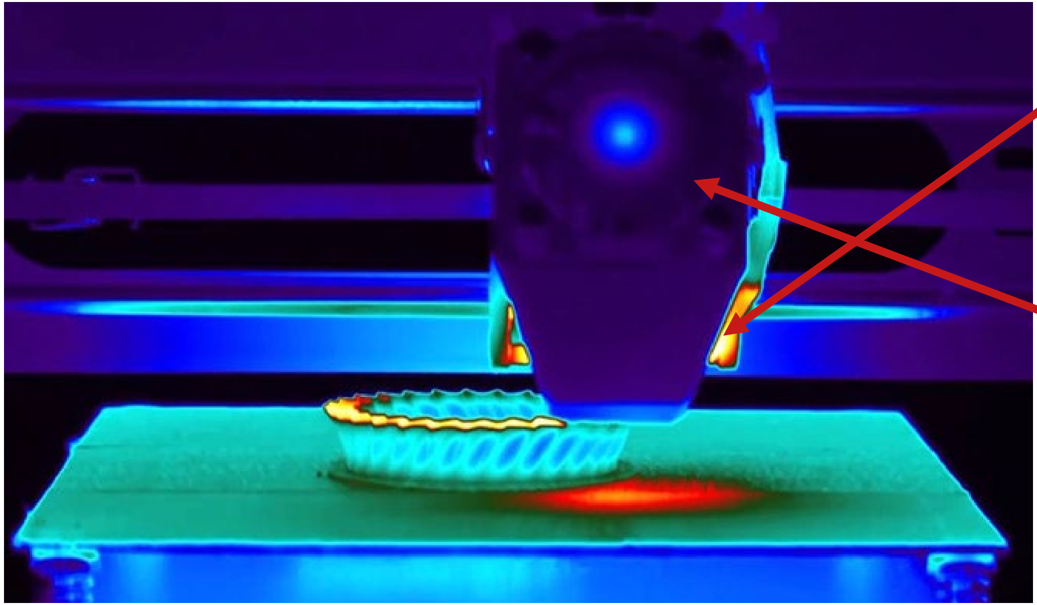
Heat transfer



If we can predict heating,
we can predict deformations.

If we can predict deformations,
we can correct positioning.

If we can correct positioning, manufacturing will be more precise.

Image: moviTHERM https://movitherm.com/blog/3d-printing-and-the-advantages-of-thermal-monitoring/

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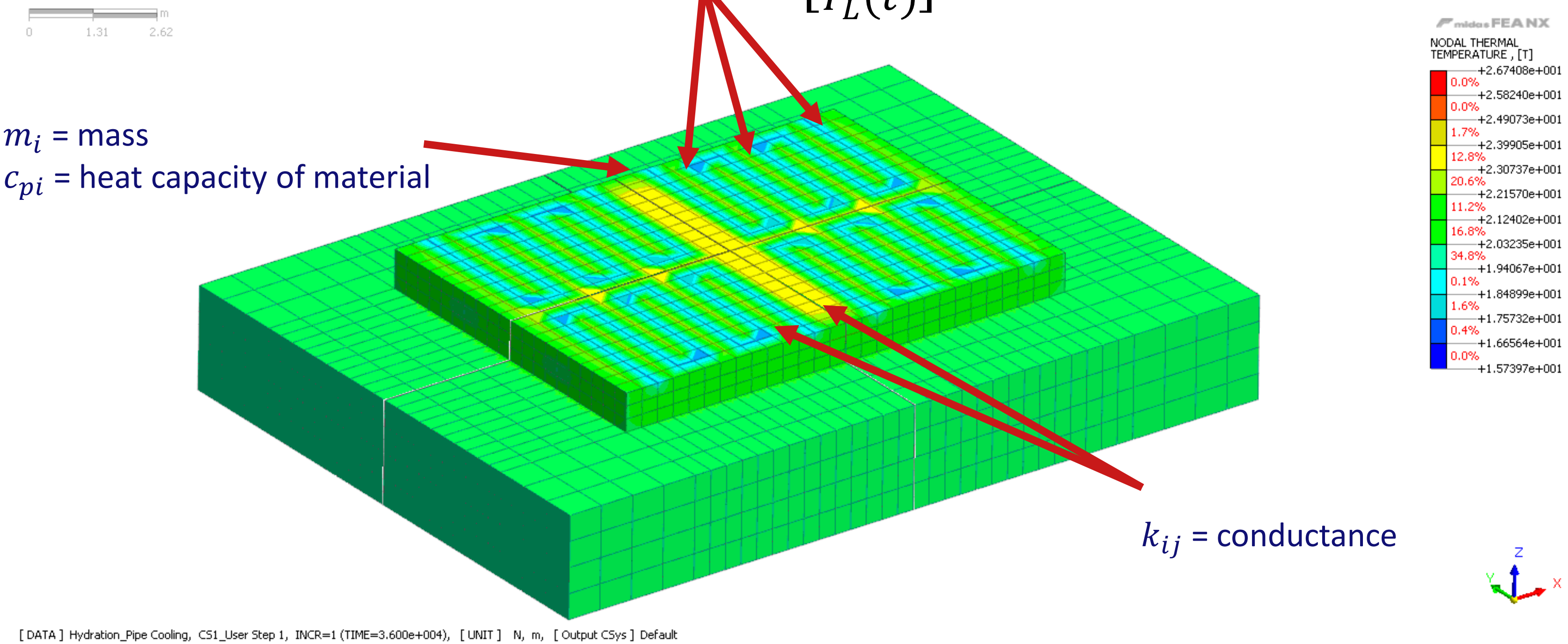
Conduction

$$T(t) = \begin{bmatrix} T_1(t) \\ \vdots \\ T_L(t) \end{bmatrix}$$

m_i = mass

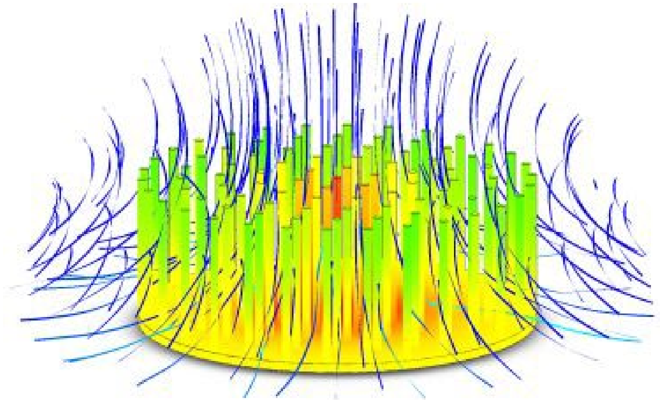
c_{pi} = heat capacity of material



k_{ij} = conductance

Image: midas FEA NX https://www.midasoft.com/bridge-library/heat-transfer-and-concrete-heat-of-hydration-analysis

Convection



Convection is exchange of heat with the medium around the machine.

- Nonlinear transient.
- Reaches steady-state over time.

Modelling convection typically requires computational fluid dynamics solvers.

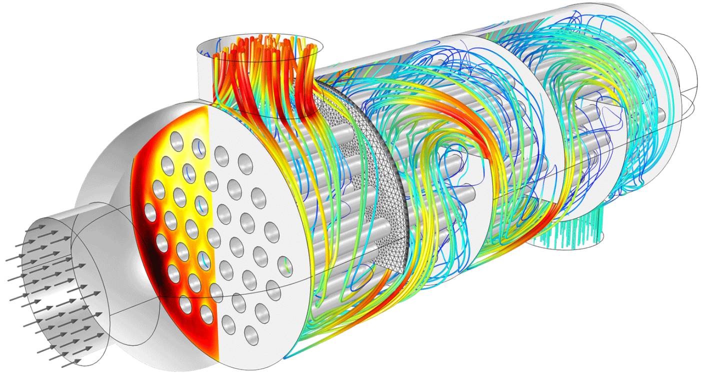


Image: https://commons.wikimedia.org/wiki/File:Natural-convection-heat-sink-fluid-WBG.jpg

Image: COMSOL https://www.comsol.com/blogs/conjugate-heat-transfer

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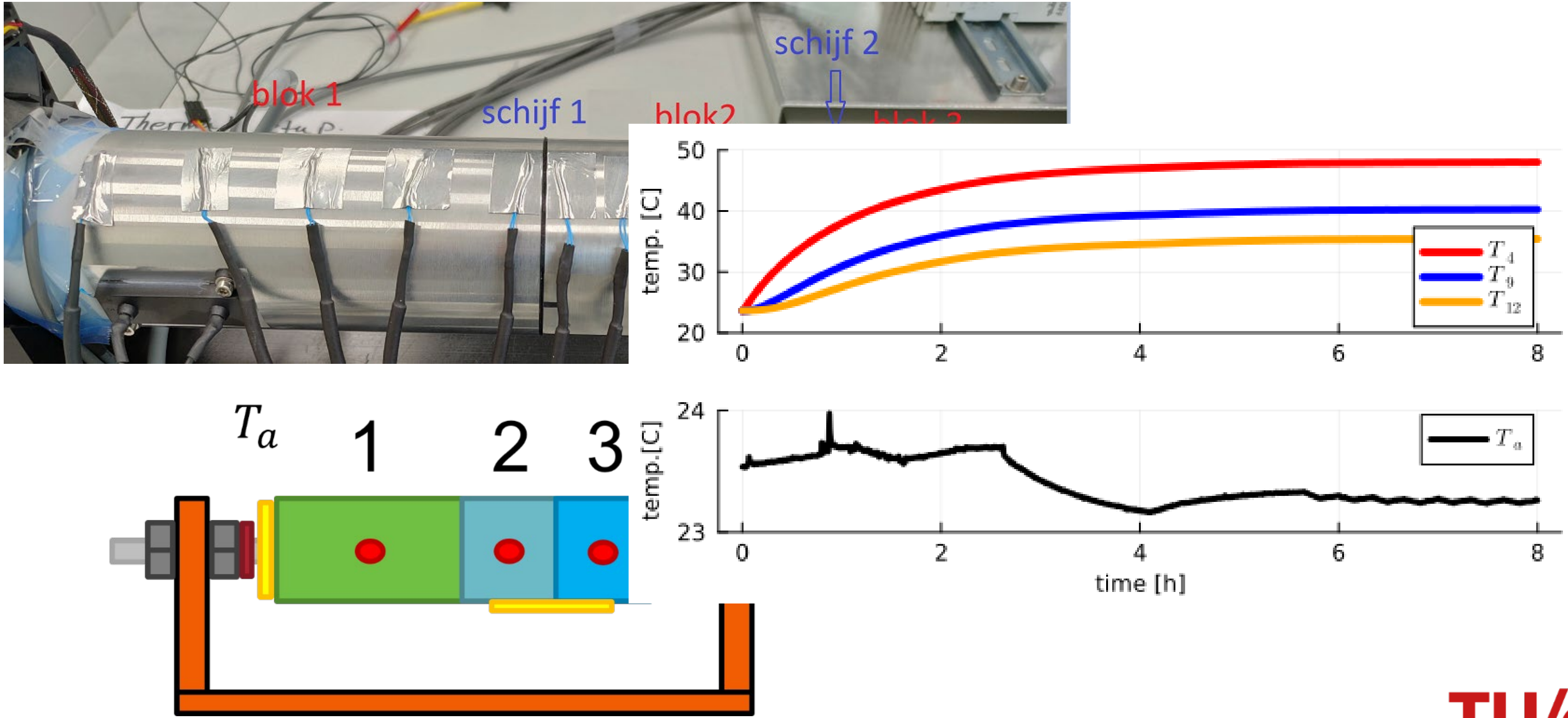
Demonstrator: heated rod



Dynamics

Lumped-element model:

cell temperatures

$$T(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \\ T_3(t) \end{bmatrix}$$

$$M\dot{T}(t) = KT(t) + h\big(T(t), T_a(t)\big) + u(t)$$

$$u(t) = \begin{bmatrix} u_1(t) \\ 0 \\ 0 \end{bmatrix}$$

heat input

ambient temperature

$$M = \begin{bmatrix} m_1 c_{p1} & 0 & 0 \\ 0 & m_2 c_{p2} & 0 \\ 0 & 0 & m_3 c_{p3} \end{bmatrix}$$

m_i = mass of each cell
c_{pi} = specific heat capacity

$$K = \begin{bmatrix} -k_{12} & k_{12} & 0 \\ k_{12} & -(k_{12} + k_{23}) & k_{23} \\ 0 & k_{23} & -k_{23} \end{bmatrix}$$

conductances

Dynamics

Convection can be split into linear and nonlinear components:

$$h(T_i(t), T_a(t)) = h_a a_i (T_a(t) - T_i(t)) + r(T_i(t), T_a(t))$$

convection coefficient (h_a)

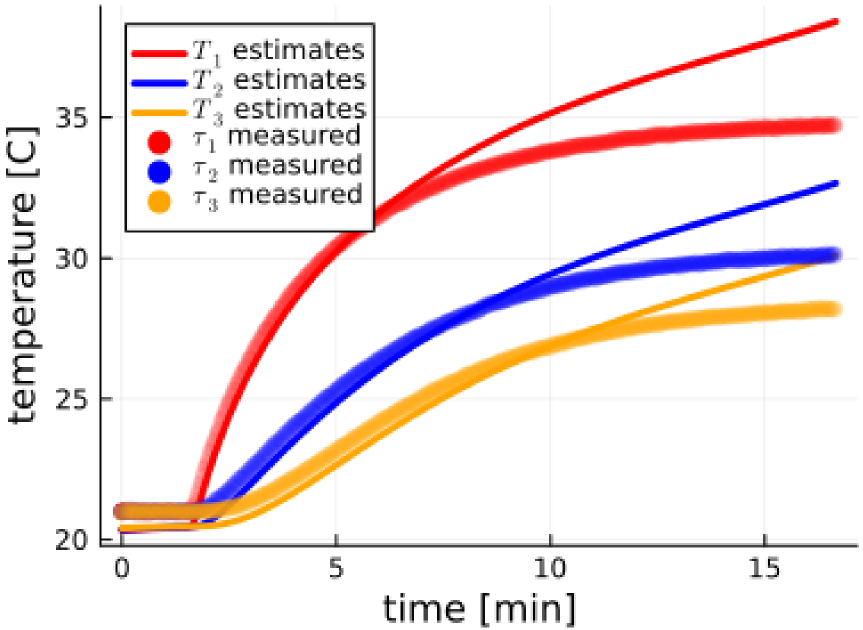
surface area of cell (a_i)

Combining linear components and absorbing ambient temperature into input, yields:

$$M\dot{T}(t) = FT(t) + r\big(T(t), T_a(t)\big) + G\bar{u}(t)$$

$$F = K - \begin{bmatrix} h_a a_1 & 0 & 0 \\ 0 & h_a a_2 & 0 \\ 0 & 0 & h_a a_3 \end{bmatrix} \qquad G = \begin{bmatrix} h_a a_1 & \\ h_a a_2 & I \\ h_a a_3 & \end{bmatrix} \qquad \bar{u}(t) = \begin{bmatrix} T_a(t) \\ u_1(t) \\ 0 \\ 0 \end{bmatrix}$$

Simulation without nonlinear convection effects



Approximate convection effects

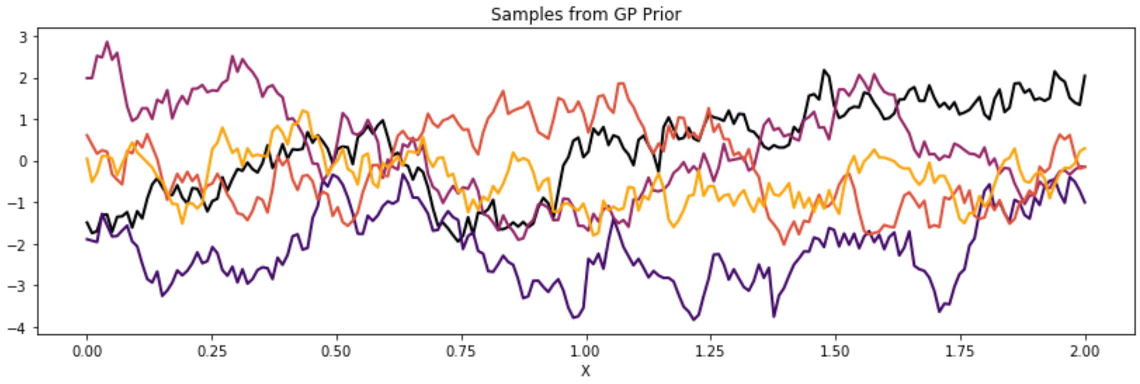
The effect of convection on temperature change can be estimated with a GP-SSM:

$$r\big(T(t), T_a(T)\big) \approx \rho(t)$$

where

$$\dot{\rho}(t) = -\lambda\rho(t) + w(t)$$

This SDE corresponds to a Gaussian process with kernel covariance function:



Model specification

The temperature dynamics model can be augmented with the GP SDE:

$$\begin{bmatrix}\dot{T}(t)\\ \dot{\rho}(t)\end{bmatrix} = \begin{bmatrix}M^{-1}F & M^{-1}\\ 0 & -\lambda I\end{bmatrix}\underbrace{\begin{bmatrix}T(t)\\ \rho(t)\end{bmatrix}}_{x(t)} + \begin{bmatrix}M^{-1}G\\ 0\end{bmatrix}\bar{u}(t) + \begin{bmatrix}0\\ 1\end{bmatrix}w(t)$$

Discretizing and casting to probabilistic form gives:

$$p(x_k|x_{k-1},\bar{u}_k) = \mathcal{N}(x_k|Ax_{k-1} + B\bar{u}_k, Q)$$

$$x_k = \begin{bmatrix}T_{k1}\\ \vdots\\ T_{kL}\\ \rho_{k1}\\ \vdots\\ \rho_{kL}\end{bmatrix}$$

We add a likelihood term for the noisy temperature observations:

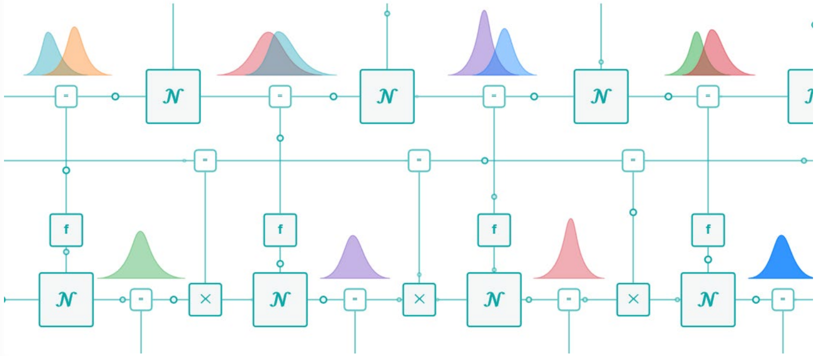
$$p(y_k|x_k) = \mathcal{N}(y_k|Cx_k, R)$$

Inference

State estimation using Bayesian filtering / smoothing: $p(x_k | y_{1:N}, \bar{u}_{1:N}) = \mathcal{N}(x_k | m_k, S_k)$

```
@model function SSM(y,u, A,B,C,Q,R,m0,S0,T)

    # State prior distribution
    x_0 ~ MvNormal(m0, S0)

    x_kmin1 = x_0
    for k = 1:T

        # Stochastic state transition
        x[k] ~ MvNormal(A*x_kmin1 + B*u[k], Q)

        # Likelihood function
        y[k] ~ MvNormal(C*x[k], R)

        x_kmin1 = x[k]
    end
end
```

Automatic Bayesian Inference through **Reactive Message Passing**



Inference

State estimates only capture the *effect* of convection on the temperature gradient.

- We want to know the *function* between temperatures T_{ik}, T_{ak} and convection effect ρ_{ik}.
- We want to quantify the uncertainty around this function estimate.

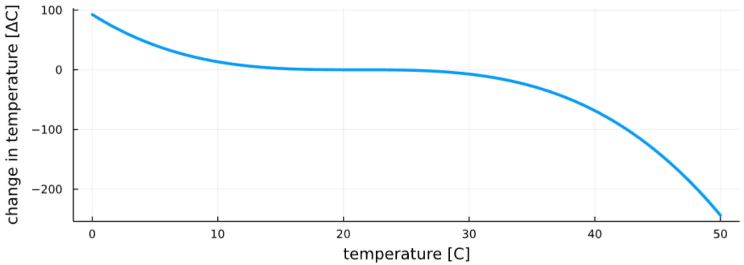
Solution:

Bayesian regression of T_{ak} and MAP estimates of T_{ik} onto distribution of ρ_{ik}.

$$p(m_{jk} | m_{ik}, T_{ak}, \theta_i) = \mathcal{N}(m_{jk} | \theta_i^\intercal \varphi(m_{ik}, T_{ak}), S_{jjk})$$
$$p(\theta_i) = \mathcal{N}(\theta_i | 0, W_0^{-1})$$

Experiment: verification

Experiment uses designed polynomial nonlinear convection function:



Goal: recover nonlinear convection function.

Identification:

1. State estimation from time series (Bayesian smoothing).
2. Regress temperature states onto ρ states (Bayesian polynomial regression).

Experiment: verification

Evaluation consists of comparing simulated temperatures between true and identified system.

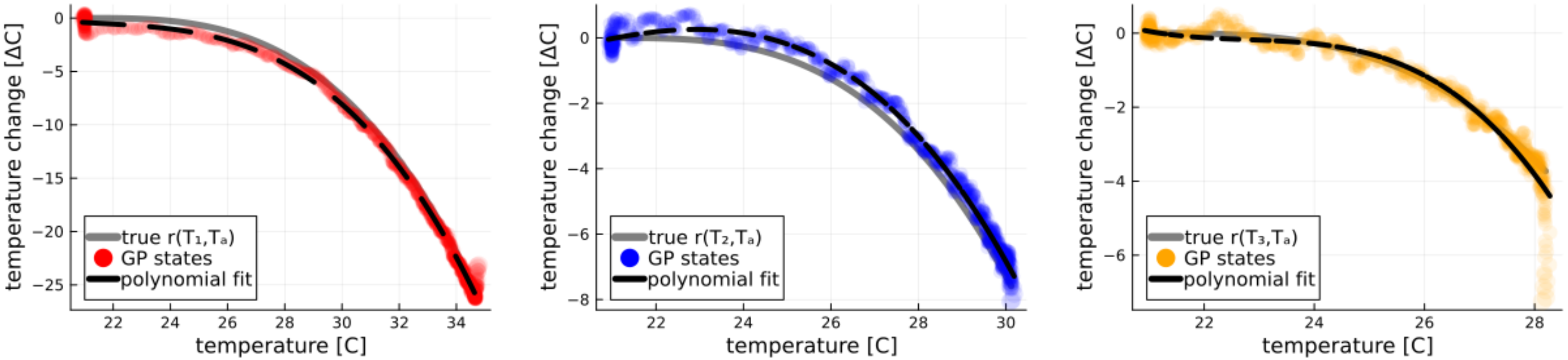
True system:

- Evolve temperatures using dynamics model + designed nonlinear convection function.

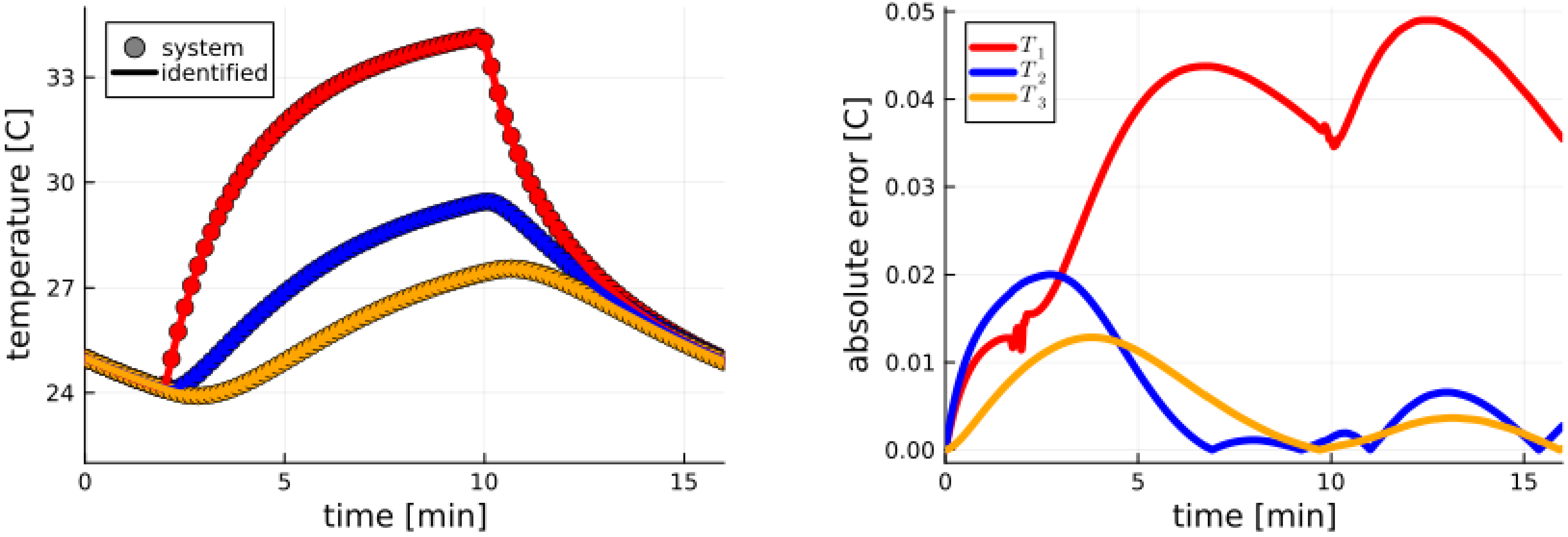
Identified system:

- Evolve temperature forward using dynamics model + identified polynomial regression function (using current temperature estimate and ambient temperature).

Experiment: verification



Experiment: verification



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Experiment: validation

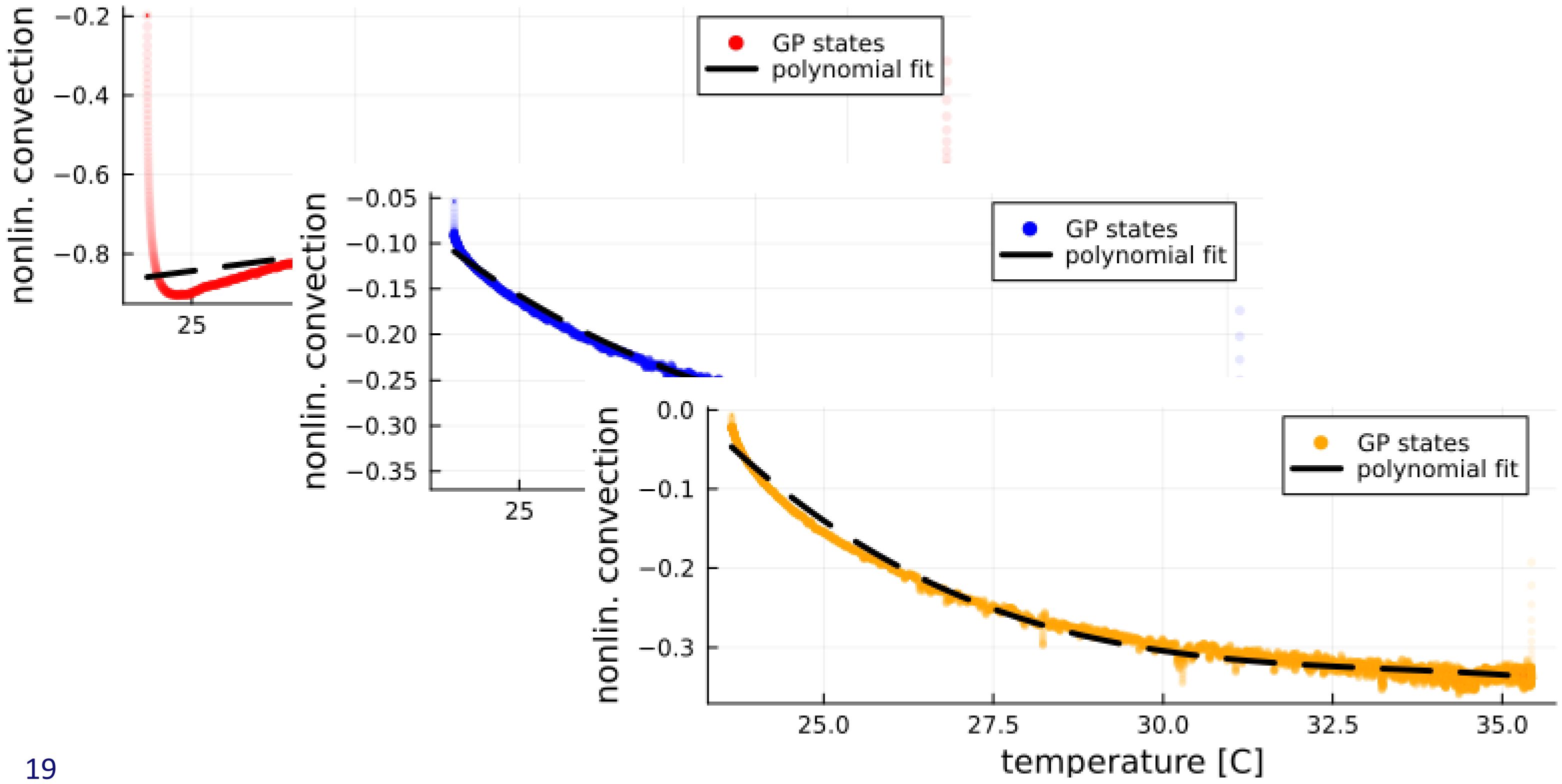
Identification:

1. State estimation from observations (Bayesian smoothing).
2. Regress temperature states and ambient temperature onto ρ states (Bayesian regression).

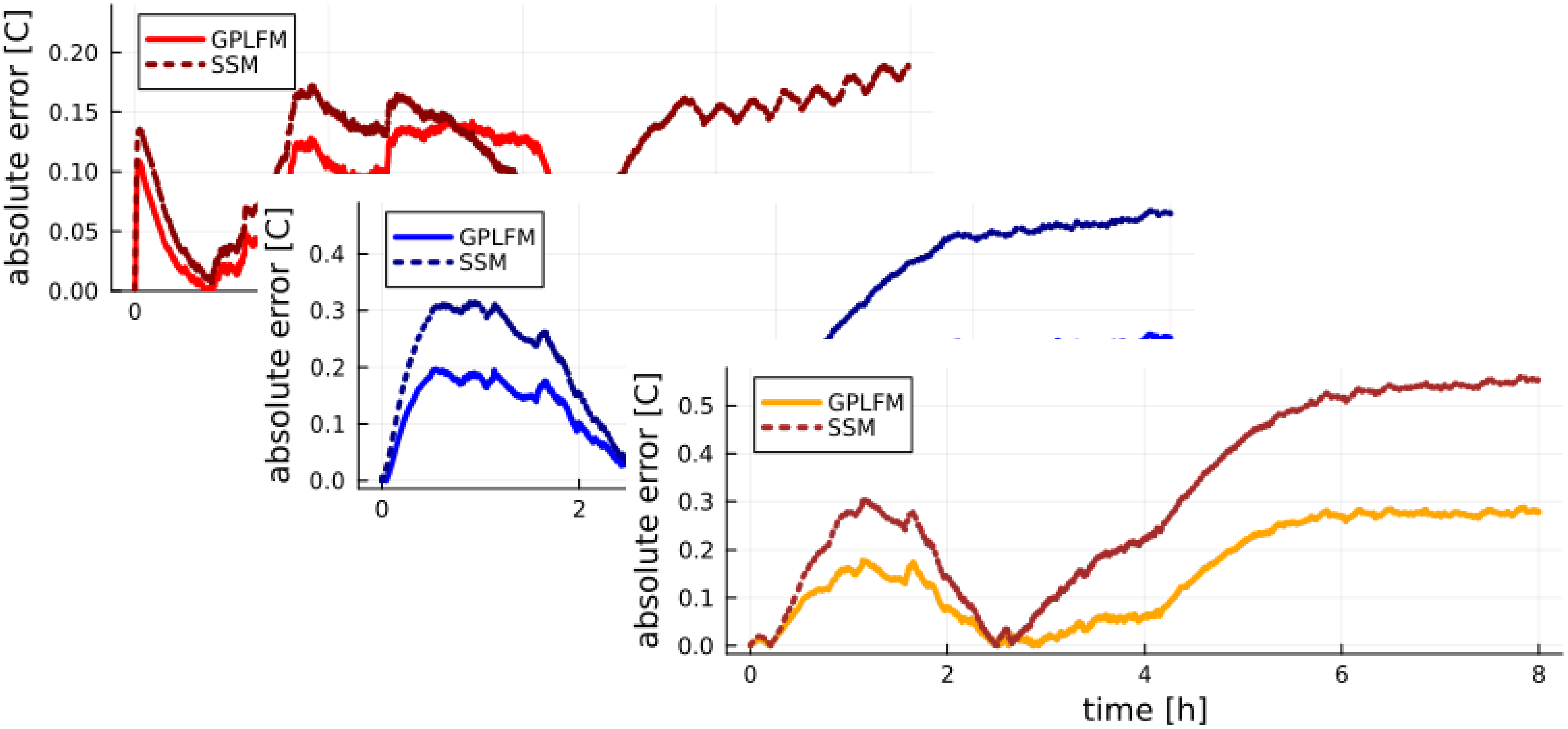
Evaluation by simulation:

1. Evolve temperature using dynamics model.
2. Evolve temperature using dynamics model + polynomial regression function applied to current temperature and ambient temperature.
3. Compare difference between simulated and observed temperatures.

Experiment: validation



Experiment: validation



Discussion

Limitations:

- Currently a two-stage procedure (online + offline).
  - But regression coefficients could be estimated recursively.

- Every cell requires has 1 accompanying GP-SSM, thereby doubling the number of states.
  - For higher-order Whittle-Matérn kernels, this becomes a tripling or quadrupling.

Future work:

- Group Gaussian processes over temperature cells.
- Co-optimization with conduction parameter estimation.
- Modelling effects of active convection.

Conclusion

- Heat transfer dynamics can be augmented with Gaussian processes in SDE form.
- Gaussian process latent force models can recover convection effects in heat transfer.
- Gaussian process latent force models are computationally cost-effective (online estimation).

Thank you

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