EINDHOVEN UNIVERSITY OF TECHNOLOGY

BIASlab

ONLINE SYSTEM IDENTIFICATION IN A DUFFING OSCILLATOR USING FREE ENERGY MINIMISATION

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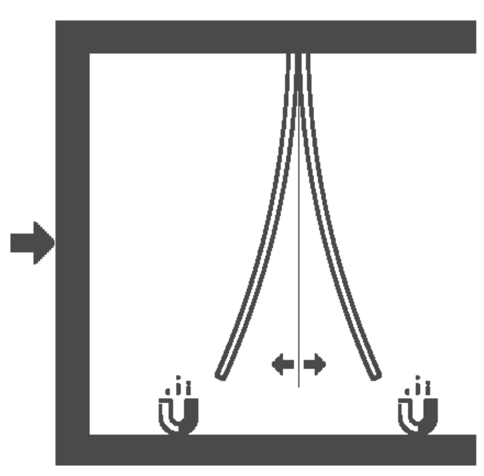
I investigate free energy minimisation, in the form of variational message passing on Forney-style factor graphs, for online system identification. To be precise, I take an example, the Duffing oscillator, and cast its differential equation to a generative model. Using a factor graph representation, I infer states and parameters with message passing. A simulation experiment, where the model predicts future output, indicates this procedure performs on par with the state-of-the-art.

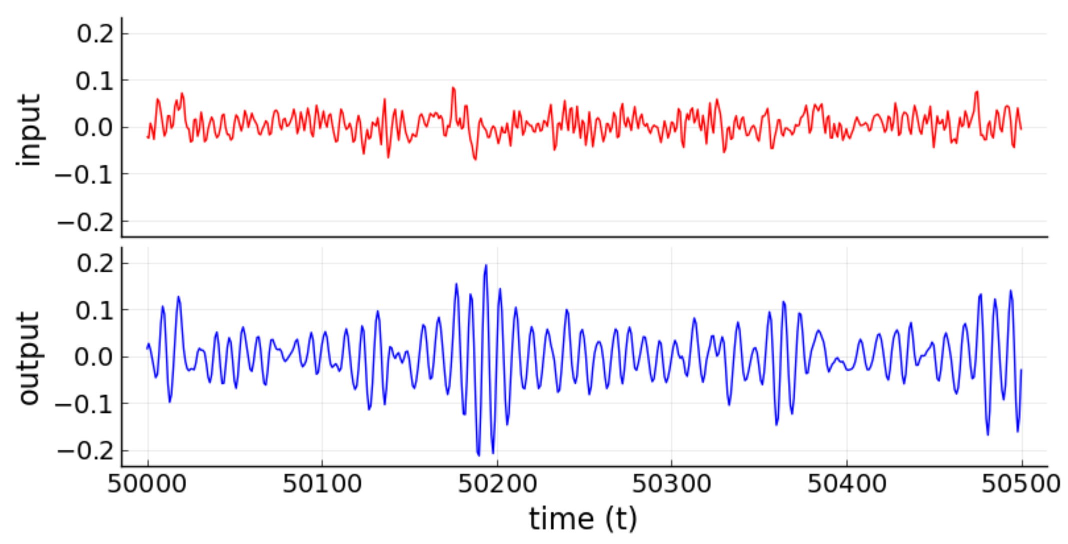
ONLINE SYSTEM IDENTIFICATION

Online system identification is the estimation of parameters of a dynamical system, such as mass or friction coefficients, for each measurement of the input and output signals. The goal is to use these inferred parameters to predict the output of the signal as well as possible.

DUFFING OSCILLATOR

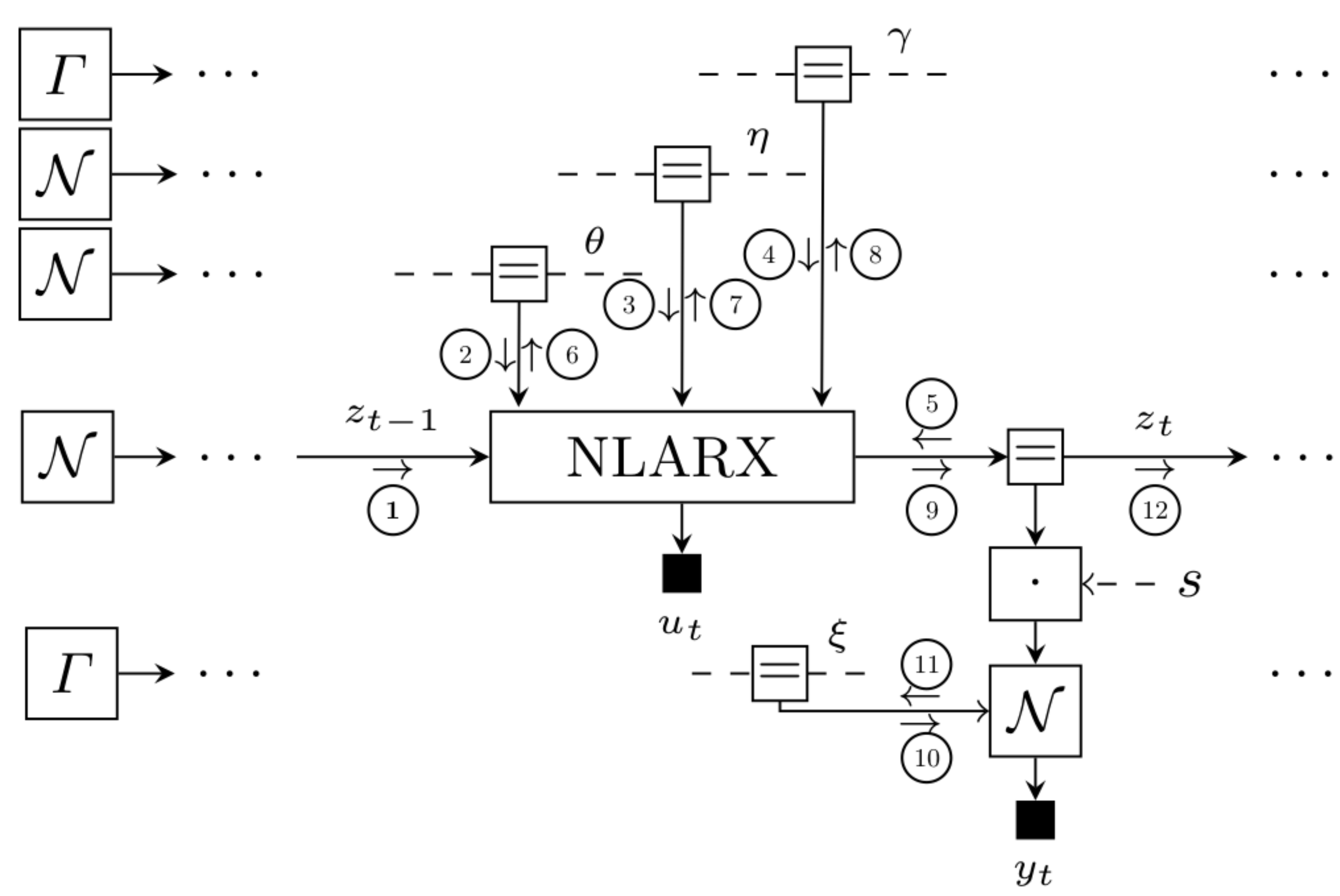
A Duffing oscillator is a driven damped harmonic oscillator with a cubic nonlinear spring coefficient. The left figure below shows an example of a physical Duffing oscillator: a steel beam is attached to the top prong of a rigid frame. Driving the frame produces a horizontal displacement in the beam, that is modulated by two magnets attached to the bottom prong. The figure on the right shows an example of the driving force (input signal) and the observed displacement (output signal).





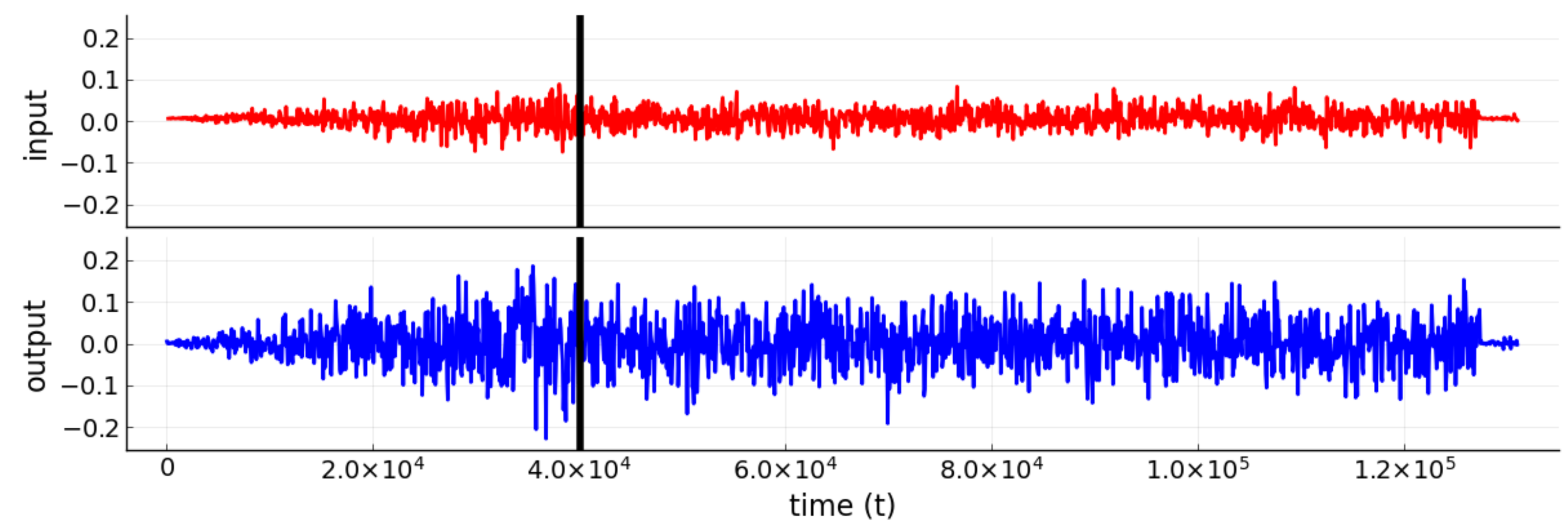
MODEL IN FORNEY FACTOR GRAPH FORM

The generative model of the Duffing oscillator is cast to a factor graph in which I perform message passing. The figure below shows the subgraph that is recursively applied over time. The terminal nodes on the left represent initial priors. They start passing messages which arrive at the state transition node, marked NLARX. There, the current beliefs for parameters, the previous state and the current observation are combined to produce a belief for the current state.



SIMULATION EXPERIMENT

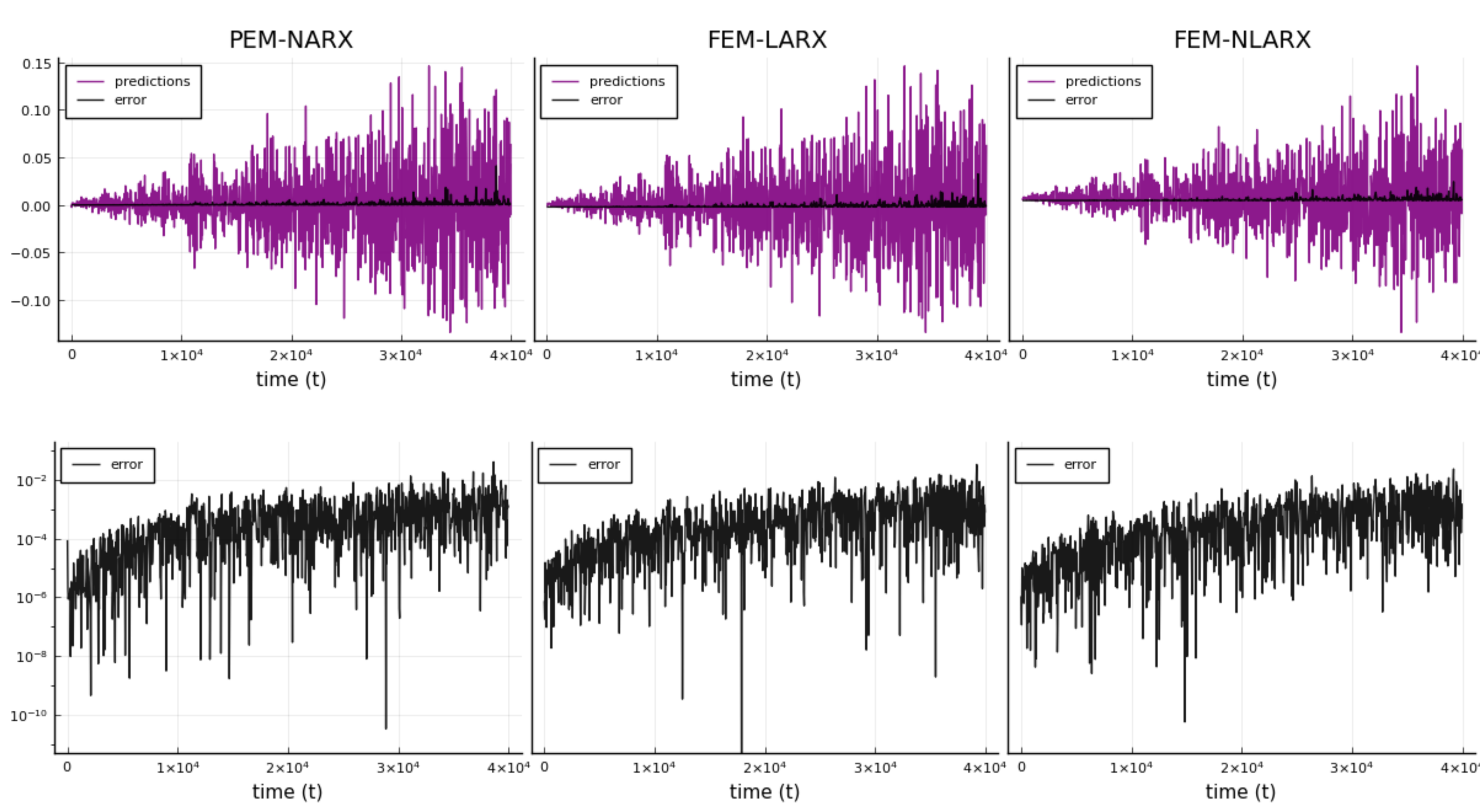
The Nonlinear Benchmark platform (http://nonlinearbenchmark.org/#Silverbox) contains a data set of an electronic implementation of a Duffing oscillator. I split the set in two: the first regime, containing 40 000 samples (fs = 610.35 Hz, so ~65 seconds) with an increasing amplitude in the input, act as a validation set. The second regime, containing the remaining 91 072 samples with odd harmonics only, acts as a training set. At training time, the model gets both input and output and infers parameters. At test time, it is given the first two samples of the validation set and must predict all output from there onwards, using the inferred parameters and the input signal.



The full model using free energy minimisation was called FEM-NLARX, and the linear variant was called FEM-LARX. I compared their predictions to that of a Nonlinear AutoRegressive model with eXogenous input (NARX) trained offline with Prediction Error Minimisation, which I called PEM-NARX.

Results

The top row in the figure below shows the predictions of all three models (purple) and their squared error (black). Note that the errors increase as the input signal's amplitude rises. The bottom row plots the errors on a log-scale. PEM-NARX has a mean squared error of 1.000e-3, FEM-LARX one of 1.002e-3 and FEM-NLARX one of 0.926e-3.



DISCUSSION

The experimental results suggest that free energy minimisation, in the form of variational message passing, is well suited to online system identification. The difficulties mostly lie in deriving variational messages. Improvements could be made with a hierarchy of latent Gaussian filters or autoregressive processes to obtain time-varying noise parameters or time-varying coefficients. Furthermore, instead of discretising to obtain an auto-regressive model, one could express the evolution of the states in generalised coordinates. Lastly, black-box models could be explored for further performance improvements.

The next step in the project is to infer the input signal regime that leads to the most accurate parameter estimates as fast as possible (i.e. optimal design). The plan is to switch from variational free energy minimisation to expected free energy minimisation, turning the model into an active inference agent.

https://github.com/biaslab/IWAI2020-onlinesysid
https://arxiv.org/abs/2009.00845
https://wmkouw.github.io/